Wind Turbine Sliding Mode Control and
Wind Farm Energy Optimization with Fatigue Constraints

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Abstract: Maximizing energy capture during wind turbine operation plays a crucial role in increasing wind energy economic viability and its penetration in the grid. When a single wind turbine is considered, the main control objective is to maximize the captured power, optimizing its power conversion efficiency, reducing mechanical fatigue and attenuating the output chattering. A super-twisting sliding mode controller is proposed, which is robust to uncertainties of turbine and generator, as well as to electric grid disturbances. The sliding mode controller designs a continuous torque and improves the wind turbine performance by enhancing energy capture and reducing dynamic loads. In a wind farm turbines are close, interferences among them can lead to outage of wind turbines. The operational points of each turbine in the wind farm in order to get interferences attenuation are found by solving the proposed optimization problem.

Keywords: Wind Farm, Fatigue reduction, Sliding Mode Control

1. INTRODUCTION

Wind power plays an important role in supporting the power grid. World wind power generation has increased more than ten times during the last decade [1]. However, significant progress is needed in the production of wind power in order to increase its economic viability and boost its penetration into the electric grid. With the use of variable speed operation, wind turbine works more efficiently in the partial load region, also defined as Region 2 [2, 3]. Moreover, wind turbines are complex nonlinear systems with uncertain parameters, and affected by unmodeled dynamics and unknown disturbances.

The objective of this paper is to control the power generated by both a single wind turbine and a wind farm, in which the set-point is defined by an optimization problem. For a single wind turbine, the main control objective is to maximize the captured power, optimizing its power conversion efficiency, reducing mechanical fatigue and attenuating the output chattering. A super-twisting sliding mode controller [4], sliding mode controller [5], is proposed, because this control strategy presents attractive features such as robustness to parametric uncertainties of the turbine and the generator as well as to electric grid disturbances. Moreover, the STW sliding mode approach designs a continuous torque, thus reducing the chattering phenomenon and therefore the mechanical stress since no strong torque variations are required.

In literature different control algorithms are proposed for Region 2 operation, starting from traditional output feedback control [6], optimal control [7] or adaptive control [8]. In [8], different operating points are analyzed, instead of many works have proposed controllers to work around an operating point using control of the generator torque to keep the turbine at a condition of maximum power point tracking (MPPT). The efficiency of the wind power conversion systems can be greatly improved using an appropriate control algorithm. In the last five years, different control schemes, based on the theory of sliding mode control (SMC), are presented. In [9], an adaptive robust control for a variable speed wind power generation is described. A robust aerodynamic torque observer is also designed in order to avoid the wind speed sensors. The proposed adaptive robust control law is based on a sliding mode control theory, that presents a good performance under system uncertainties.

For the wind farm, the main control objective is related to “a power” smoothing [10], to reduced fatigue loads [11], and to minimize load [12]. As clearly described in [2], the MPPT is used to compensate for unknown or time-varying parameters, which are sometimes the cause of poor efficiency for a wind farm. Due to the aerodynamic interaction, the strategy of having each wind turbine in an array extract as much power as possible does not lead to maximal total overall power capture across the entire array, [2, 13].

The paper is organized as follows. In Section 2 the dynamical model of a single wind turbine is presented. Section 3 proposes the STW sliding mode controller designed for a single wind turbine, for which the rotor speed reference is evaluated to track the maximum power point. The overview of a wind farm main features is presented in Section 4, while in the following section the model of a wind farm is briefly introduced. In Section 6 the considered optimization problem is defined; in a wind farm several turbines are located and operate close, therefore interferences among them raise and must be considered.

† Takayuki Wada is the presenter of this paper.
In fact these interactions can lead to outage of the wind turbines. The operation points of each turbine in the wind farm in order to get interferences attenuation can be found by solving the optimization problem. Finally, some concluding remarks are described in Section 7.

2. WIND TURBINE MODEL

First, a single turbine is considered. Wind turbines are designed to produce electrical energy as cheaply as possible. All wind turbines are therefore designed with some sort of power control. The designed control law keeps the turbine operating at the peak of its power coefficient, which could be correlated with the thrust coefficient $C_T$ ($P = T\omega$, where $P$ is the power, $T$ is the thrust and $\omega$ is the angular velocity).

The wind turbine should be driven according to the following three fundamental modes (regions) associated with wind speed, maximum allowable rotor speed, and rated power (see Figure 1 and [14]):

Region 1: operating at variable speed/variable tip-speed ratio.

Region 2: operating at variable speed/variable tip-speed ratio.

Region 3: operating at variable speed/constant power.

As already described in Introduction, in this paper we focus on the analysis of Region 2, that is the section in which the wind speed is approximately from 5 to 14 m/s. The main idea is that in Region 2 it is captured as much power as possible from the wind and, for most of the time, in this region, the blade pitch is constant at an optimal value for peak energy extraction.

![Fig. 1 Steady-state power curves [14]](image)

The system modeling is inspired from [15, 16] and it is simplified considering the external stiffness negligible. So, the rotor dynamics can be written as

$$\dot{\omega}_r = \frac{T_a - K_t\omega_r - T_g}{J_t},$$  \hspace{1cm} (1)

where $\omega_r$ is the rotor speed, $T_a$ and $T_g$ are the aerodynamic and generator torques, respectively. $K_t$ is the combination of the rotor and generator external damping. In a similar way, the moment of inertia $J_t$ is a combination of the rotor and generator moment of inertia. In the evaluation of these two coefficients the gearbox ratio is also considered. Moreover, these two coefficients are known and constant in each operating points. All the symbols are also explained with Fig. 2.

The aerodynamic torque $T_a$ can be evaluated starting from the evaluation of the trust coefficient $C_T$. So,

$$T_a = \frac{1}{2} \pi \rho R^3 C_T(\lambda, \beta) w^2,$$

with air density $\rho$, rotor radius $R$, and wind speed $w$. As indicated in the formula, the thrust coefficient is function of the tip-speed ratio $\lambda$ and of the pitch angle $\beta$. In our case, the pitch angle is considered constant and selected to obtain the maximum thrust coefficient. Instead, the tip-speed ratio is function of the wind speed and of the rotor angular velocity, that is $\lambda = \frac{R\omega_r}{w}$.

![Fig. 2 Wind turbine dynamics [16]](image)

For the optimization problem, we focus on the variation of the power coefficient. Starting from the thrust evaluation, we write that

$$P_a = T_a\omega_r,$$

$$P_g = T_g\omega_r,$$

where $P_a$ is the aerodynamic power and $P_g$ is the generator power.

In this way the power coefficient variation with $\lambda$ and $\beta$ can be evaluated as

$$C_p(\lambda, \beta) = 0.22 \left(\frac{116}{\lambda} - 0.4\beta - 5\right) e^{-\frac{12.5}{\lambda_i}},$$ \hspace{1cm} (2)

with $\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} = \frac{0.035}{\beta^3 + 1}$. The pitch angle is in degree.

For the designed control problem, we need to evaluate an optimum point, to obtain the maximum power. For this reason, the optimum point is evaluated from the maximum angular rotation speed $\omega_{opt}$. We can obtain this angular speed starting from

$$\omega_{opt} = \frac{\lambda_{opt} w}{R},$$ \hspace{1cm} (3)

and evaluating $\lambda_{opt}$ searching for the peak of power coefficient, in which $\beta$ is considered constant and fixed. A variable wind speed behavior is included in the model starting from the model described in [17] and including random noise.
3. SLIDING MODE CONTROL FOR WIND TURBINE

In this section the proposed super-twisting (STW), \cite{4}, sliding mode control strategy for the wind turbine is presented. This strategy is based on a second order sliding mode controller. Indeed, sliding mode control is well known to be an effective nonlinear robust control approach, which can guarantee the invariance property to matched uncertainties for the controlled system’s dynamics. Furthermore, for wind turbine control, STW SMC results to be easy to implement, guarantees efficiency and designs a continuous torque.

Mitigating turbine torque variation, thus reducing the fatigue loading of a turbine, is also considered in the control design.

The main control objective is to maximize the captured power, optimizing its power conversion efficiency, reducing mechanical fatigue and attenuating the output chattering. The control of the rotor angular speed can improve the wind turbine performance by enhancing energy capture and reducing dynamic loads.

The proposed continuous sliding mode controller is robust to uncertainties of turbine and generator, as well as to electric grid disturbances.

The STW sliding mode controller designs a continuous torque, thus reducing the chattering phenomenon and therefore the mechanical stress since no discontinuous torque variations are required.

The proposed sliding mode controller is continuous and guarantees the MPPT. In particular, the STW sliding mode controller for the wind turbine (1) is designed as follows

\[
T_g = -A_1 |\omega_r - \omega_{opt}| ^ \frac{1}{2} \text{sgn}(\omega_r - \omega_{opt}) + x, \\
\dot{x} = -A_2 \text{sgn}(\omega_r - \omega_{opt}),
\]

where \( A_1 \) and \( A_2 \) are the control gains to be chosen so to counteract uncertainties and disturbances, \cite{4}, and the rotor speed reference \( \omega_{opt} \) is evaluated to track the maximum power point, corresponding to the value \( \lambda_{opt} \), according to (3).

The power reference is generated by a MPPT algorithm, which searches for the peak power on the power-speed curve, Fig. 1.

The proposed control scheme is reported in Fig. 4, in which the SMC is designed for the control of the rotor speed \( \omega_r \) and the pitch angle \( \beta \) is considered constant. As indicated in Section 2, the optimum angular speed \( \omega_{opt} \) is function of the variable wind speed and of the optimum tip-speed ratio.

The STW sliding mode controller designs a continuous torque, thus reducing the chattering phenomenon and therefore the mechanical stress since no discontinuous torque variations are required.

Now, let us consider a wind farm with \( n \) wind turbines. Wind turbines are often located with other turbines in wind farms to reduce costs by taking advantage of economies of scale. As detailed in \cite{2}, turbines on wind farms can be located along a single line, in multiple lines, in clusters, in grids, and in nearly any configuration imaginable based on geographical features, prevailing wind direction, and other factors. These other factors may include

- access requirements,
- turbine noise,
- environmental effects,
- safety, and
- visual impacts.

In our case a simple system setup is considered to validate the optimization problem and the effectiveness of the proposed control scheme: three turbines in line are considered for the analyzed wind farm, as it will be described in Section 5.

The noise, safety, visual, and environmental effects are typically more pronounced in wind farms with multiple turbines than they are for individual turbines. From a control systems perspective, wind farm research is focused mainly on two areas: control of the electricity generated by the turbines and coordinated control of the power produced by individual turbines in the farm to minimize the negative effects of turbine aerodynamic interaction. The motivation of this paper is to design a coordinated control of the power produced by individual turbines, with a "power smoothing" by means of load and fatigue reduction.

Due to the aerodynamic interaction, coordinated control of all the turbines on a farm is important \cite{18, 19}. For a single turbine, the aim of control algorithms in modern wind turbines is to adjust the control degrees of free-
5. WIND FARM MODEL

Each wind turbine receives wind effects from its neighboring wind turbines. In wind farms, in which turbines are placed relatively close to each other, the wake effect causes a coupling among the control parameters of upstream turbines and the power productions and loads on downstream turbines. Cooperative control strategies that take into account the wake effect can be used to optimize the total power production of the wind farm.

Consider a row of \( n \) wind turbines standing in the wake of each other, in a wind field with an incoming free stream speed \( V_\infty \), as depicted in Figure 5. Briefly describing some wind farm characteristics, the turbines have power productions \( P_i \) and certain control settings \( a_i \) that influence the power production of the turbines [18]. In the example proposed in this paper, it is assumed that the control variable \( a_i \) is the axial induction factor of turbine \( i \). To perform the optimization in a local sense, the control parameters \( a_i \) can be iteratively updated using a min max optimization problem. The induced axial factor is a measure of the slowing of the wind speed between the free stream (far upwind) and the rotor plane. That is,

\[
a_i = \frac{V_i}{V_\infty},
\]

in which \( V_i \) is the rotor speed, assuming the actuator disk theory, in which the rotor is modeled as an infinitely thin disk, inducing a constant velocity along the axis of rotation (see Fig. 6 and [20]). Friction is not included.

The total power of each turbine should be easily written in function of the axial factor \( a_i \) as

\[
P_i = 2\pi R^2 V_i^3 a_i (1 - a_i)^2,
\]

in which \( V_i \) is the speed acting on the \( i \)-th wind turbine, affected by the aerodynamic interaction. The speed \( V_i \) is function of the speed acting on the first turbine (i.e. the flow speed \( V_\infty \)) and of the speed deficit \( \delta V_i \) due to the wake model. The wake model is not considered in detail in this paper but the speed acting on the \( i \)-th wind turbine is reduced with respect to the speed acting on the first wind turbine. Details on the evaluation of the speed deficit are in [13, 21].

Since the wind perturbation could lead to outage of a wind turbine, intensity of turbulence effect at \( i \)-th wind turbine is modeled [22] [23, Chapter 8] as

\[
I_{\text{eff},i}(V_i) = \left( \int_{0}^{2\pi} p(\theta \mid V_i)(I_i(\theta \mid V_i))^m d\theta \right)^{1/m}
\]

where \( I_{\text{eff},i} \) is effective turbulence intensity which corresponds to fatigue risk of the wind turbine, \( p \) is a probability density function of wind direction for a given wind speed \( V_i \), \( \theta \) is wind direction, \( V_i \) is wind velocity at wind turbine hub height, \( I_i \) is turbulence intensity combined of ambient and wake flow from wind direction, and \( m \) is Wöhler exponent which depends on material of blades of the wind turbine.

In this paper, we assume that \( p \) represents the uniform distribution on \([0, 2\pi]\), wind turbines in a row are arranged at the same interval \( d_r \), and those in a column are equally spaced and its distance is \( d_f \). Under the above assumptions, effective turbulence intensity at the \( i \)-th wind turbine is given by

\[
I_{\text{eff},i}(V_i) = \begin{cases} 
\frac{\hat{\sigma}}{V_i} & \text{if } \min\{d_r, d_f\} \geq 20R \\
\frac{\hat{\sigma}_{\text{eff},i}}{V_i} & \text{if } \min\{d_r, d_f\} < 20R,
\end{cases}
\]

where \( \hat{\sigma} \) is a characteristic ambient turbulence standard deviation,

\[
\hat{\sigma}_{\text{eff},i} = \left( (1 - 0.06|N_i|)\hat{\sigma}^m + 0.06 \sum_{j \in N_i} \sigma_T^2(C_{T,j}) \right)^{1/m},
\]

\( N_i \) is the set of wind turbines which are neighbor of the \( i \)-th wind turbine, \( \sigma_T(C_{T,j}) \) is the standard deviation of the maximum center-wake turbulence at the hub height,

\[
\sigma_T(C_{T,j}) = \frac{V_i^2}{(1.5 + \frac{0.4d_{ij}}{R\sqrt{C_{T,j}}})^2 + \hat{\sigma}^2},
\]

and certain control settings \( a_i \) are equally spaced and its distance is \( d_f \).
\( d_{ij} \) is distance between the \( i \)-th and \( j \)-th wind turbine in rotor diameters, and \( C_{T,j} \) is the thrust coefficient of the \( j \)-th wind turbine.

6. ENERGY OPTIMIZATION WITH FATIGUE CONSTRAINTS

In the previous section, we see that if wind turbines are very close, there are interferences among them. Since these interferences could lead to outage of wind turbines, we would like to determine operation points of them for these interference attenuation. Effective turbulence intensity \( I_{\text{eff}} \) depends on the ambient estimated turbulence standard deviation \( \tilde{\sigma} \) and the maximum center-wake turbulence standard deviation \( \tilde{\sigma}_{T,i} \). Since we cannot control the ambient turbulence, let us consider reduction of \( \tilde{\sigma}_{T,i} \).

The optimization problem is formulated to maximize generation power subject to constraints with respect to \( \tilde{\sigma}_{T,i} \). That is, for given small positive fatigue tolerance \( \varepsilon > 0 \), we solve

\[
\begin{align*}
\text{max} & \quad \sum_{i=1}^{n} P_i \\
\text{s.t.} & \quad \sum_{j \in N_i} \left( \tilde{\sigma}_{T,i}(C_{T,j}) \right)^m \leq \varepsilon, \quad i = 1, 2, \ldots, n.
\end{align*}
\]

Notice that generation active power \( P_i \) depends on the power coefficient \( C_{P,i} \) of the \( i \)-th wind turbine and \( C_{P,j} \) related to the thrust coefficient \( C_{T,j} \), such as

\[
C_{P,i} = 4a_i(1 - a_i)^2, \quad C_{T,i} = 4a_i(1 - a_i),
\]

where \( a_i \in [0, 1] \) is the induced axial factor of the \( i \)-th wind turbine [2]. Although the induced axial factor is not a control variable of the wind turbine, it provides a reference tip-speed ratio \( \lambda_i \) and a reference pitch angle \( \beta_i \). We therefore formulate

\[
\begin{align*}
\text{max} & \quad \sum_{i=1}^{n} P_i \\
\text{s.t.} & \quad \sum_{j \in N_i} \left( \frac{1}{1.5 + \frac{0.4d_{ij}}{R\sqrt{4a_i(1 - a_i)}} \right)^m \leq \varepsilon, \quad i = 1, 2, \ldots, n.
\end{align*}
\]

for each \( i = 1, 2, \ldots, n \). From Fig. 3, we see that there exist multiple solutions \( \lambda_i \) and \( \beta_i \) according to Equation (5). However, it is not difficult to find one solution. For example, if we give \( a_i = 0.4 \), we can find a solution \((\lambda_i, \beta_i) = (13.2066, 0.0067)\).

If we replace a pitch angle \( \beta \) and \( \lambda_{\text{opt}} \) with our optimization results \( \beta_i \) and \( \lambda_i \) in Fig. 4, we can employ our sliding mode controller and optimization as a unified framework for wind farm control.

7. CONCLUDING REMARKS

Maximizing energy capture during wind turbine operation plays a crucial role in increasing wind energy economic viability and its penetration in the grid.

In this paper first it is considered the case of a single wind turbine, when the main control objective is to maximize the captured power, optimizing its power conversion efficiency, reducing mechanical fatigue and attenuating the output chattering. The proposed super-twisting sliding mode controller is robust to uncertainties of turbine and generator, as well as to electric grid disturbances. Furthermore the considered sliding mode controller is able to design a continuous torque and to improve the wind turbine performance by enhancing energy capture and reducing dynamic loads.

The considered second control problem is the wind farm case, where several wind turbines are located and have to operate close. In a wind farm interferences among the wind turbines raise and can lead to outage of the wind turbines. An optimization problem is defined with the aim to find the operational points for the turbines in the wind farm such that overall the energy capture is maximized while the fatigue is minimized, thus attenuating the effects of the interferences among the turbines in the wind farm.

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