Application to channel equalization

Tutorial @IFAC'14: Randomized methods for analysis and design of control systems

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Channel equalization: problem formulation

Goal:

send a signal *u(t)* via a transmitter (Tx) to some receiver (Rx)

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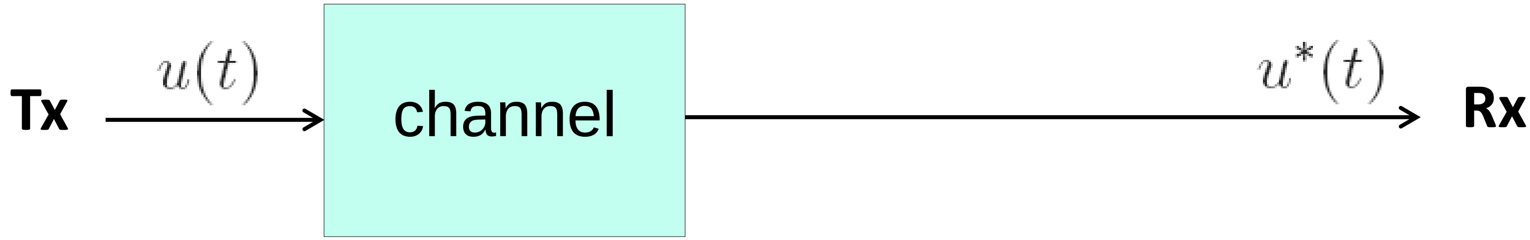

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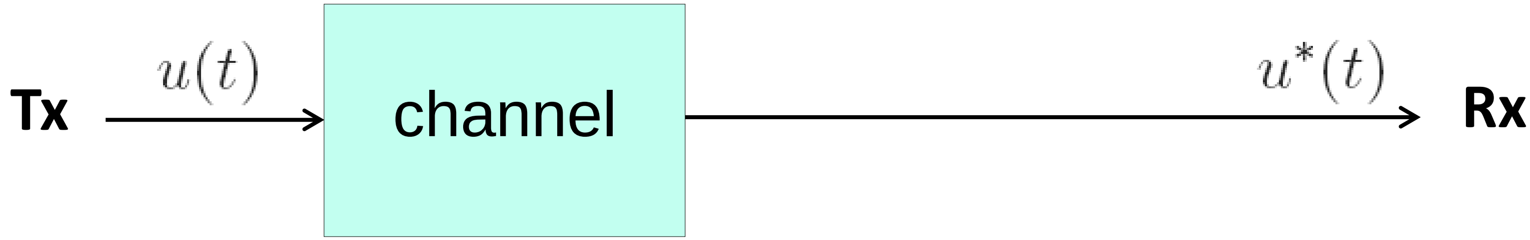

The transmission channel introduces some distortion, i.e., $u^*(t) \neq u(t)$

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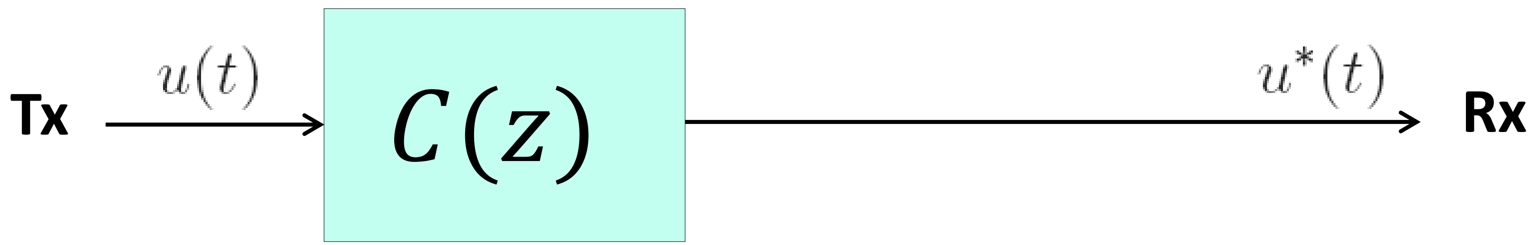

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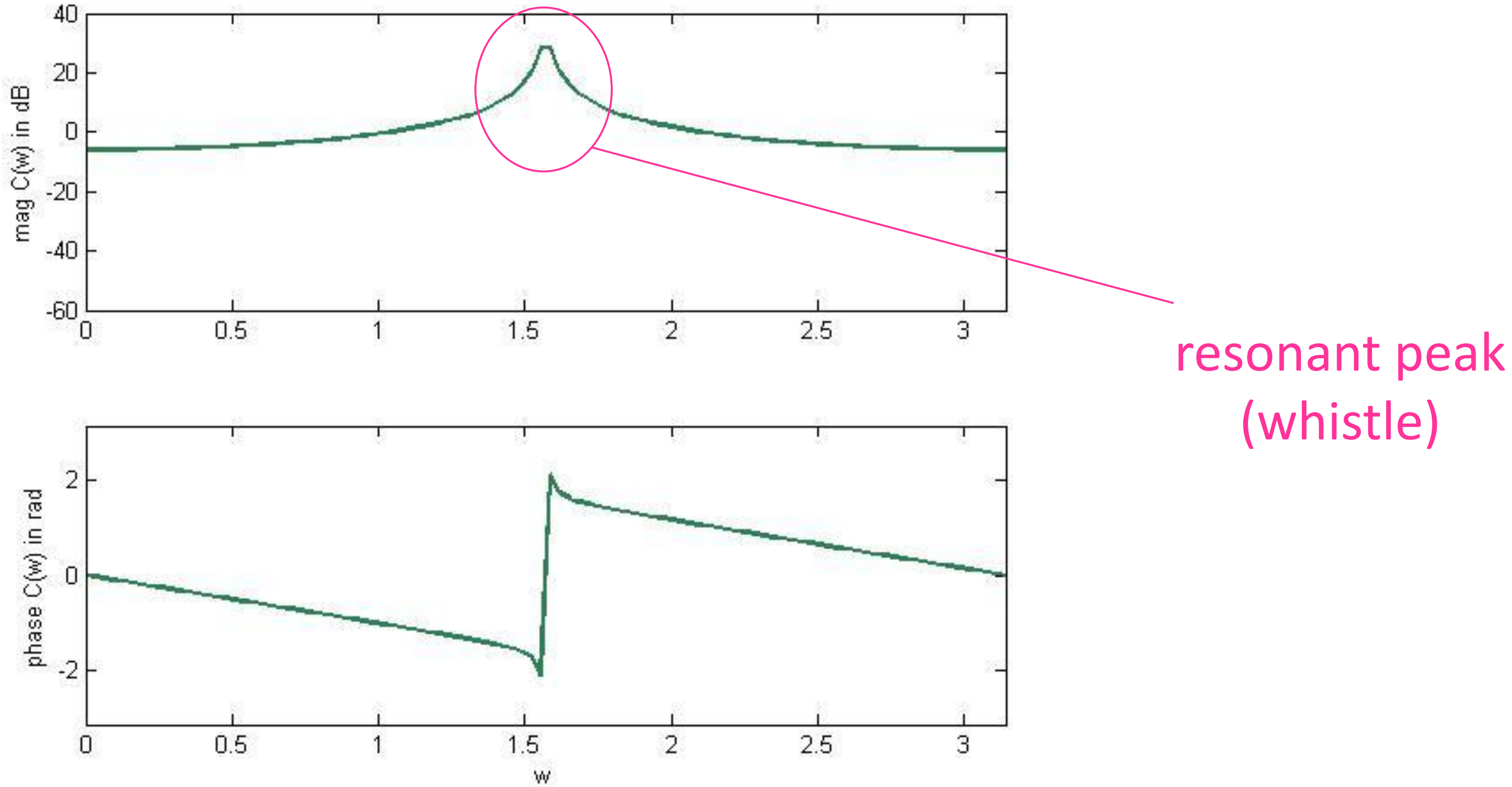
Example [distorting channel with a resonant peak]

$$C(z) = \frac{1}{z^2 + 0.98}$$

Channel equalization: problem formulation



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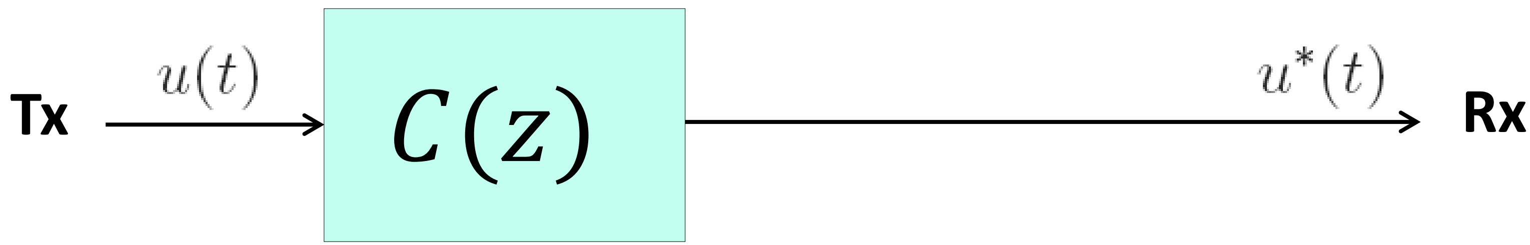

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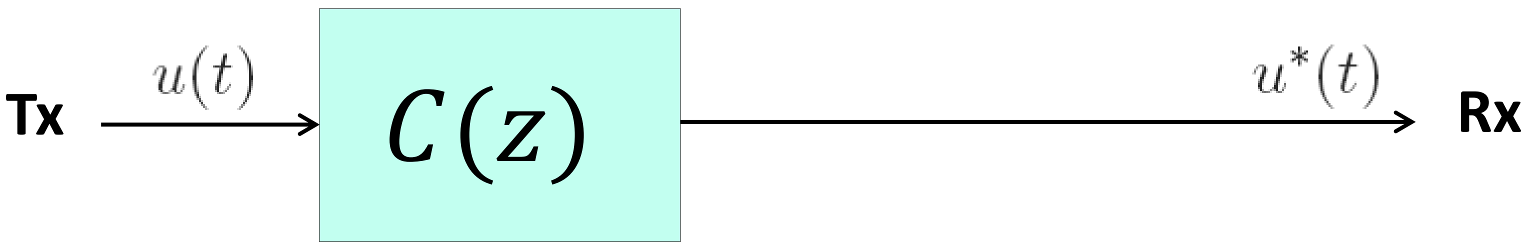

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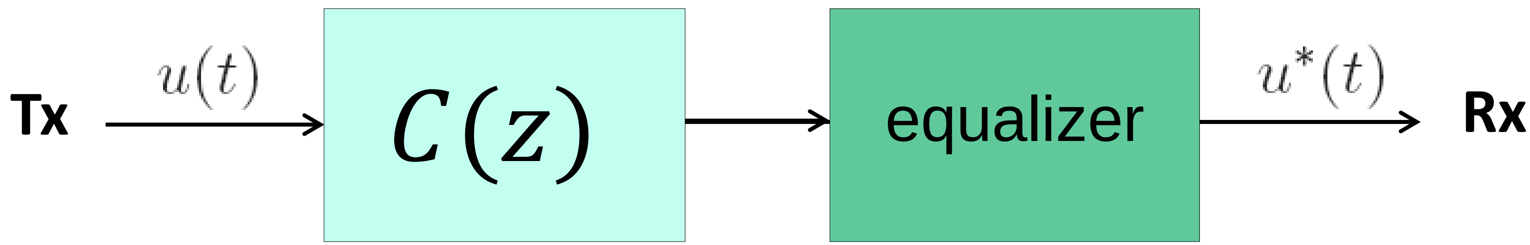

Design an equalizer so as to compensate the channel distortion

$$u^*(t) \simeq u(t-k)$$

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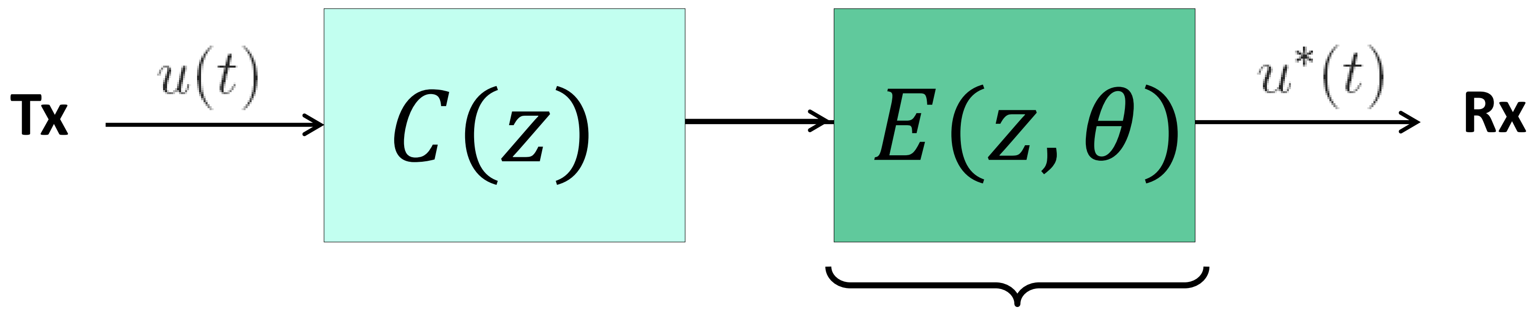

FIR equalizer: $\theta_1 + \theta_2 z^{-1} + \cdots + \theta_r z^{-r+1}$

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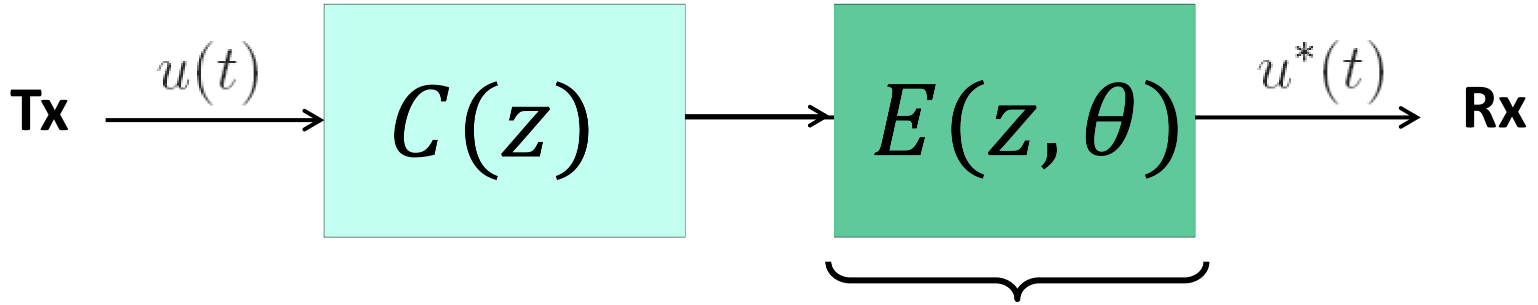

FIR equalizer: $\theta_1 + \theta_2 z^{-1} + \cdots + \theta_r z^{-r+1}$

Determine the equalizer parameter vector $\theta = [\theta_1 \ \theta_2 \ \ldots \theta_r]'$ so that

$$C(z)E(z,\theta) \simeq z^{-k}$$

Channel equalization: problem solution

Goal:

Determine the equalizer parameter vector θ so that

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Objective function

$$\|C(z)E(z,\theta) - z^{-k}\|_2^2 = \frac{1}{\pi}\int_0^{\pi} |C(e^{j\omega})E(e^{j\omega},\theta) - e^{-j\omega k}|^2 d\omega$$

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Constraint

$$\|C(z)E(z,\theta) - z^{-k}\|_\infty = \sup_{\omega} |C(e^{j\omega})E(e^{j\omega},\theta) - e^{-j\omega k}| \leq 1$$

added to detect a mismatch in a small frequency range and remove the resonant peak

Channel equalization: problem solution

Goal:

Determine the equalizer parameter vector θ so that

$$C(z)E(z,\theta) \simeq z^{-k}$$

Objective function

$$\frac{1}{m}\sum_{i=1}^{m} |C(e^{j\omega_i})E(e^{j\omega_i},\theta) - e^{-j\omega_i k}|^2$$

Constraint

$$\max_{i=1,\dots,m} |C(e^{j\omega_i})E(e^{j\omega_i},\theta) - e^{-j\omega_i k}| \leq 1$$

where $\{\omega_1, \omega_2, \dots, \omega_m\}$ = grid of $[0, \pi]$

Channel equalization: problem solution

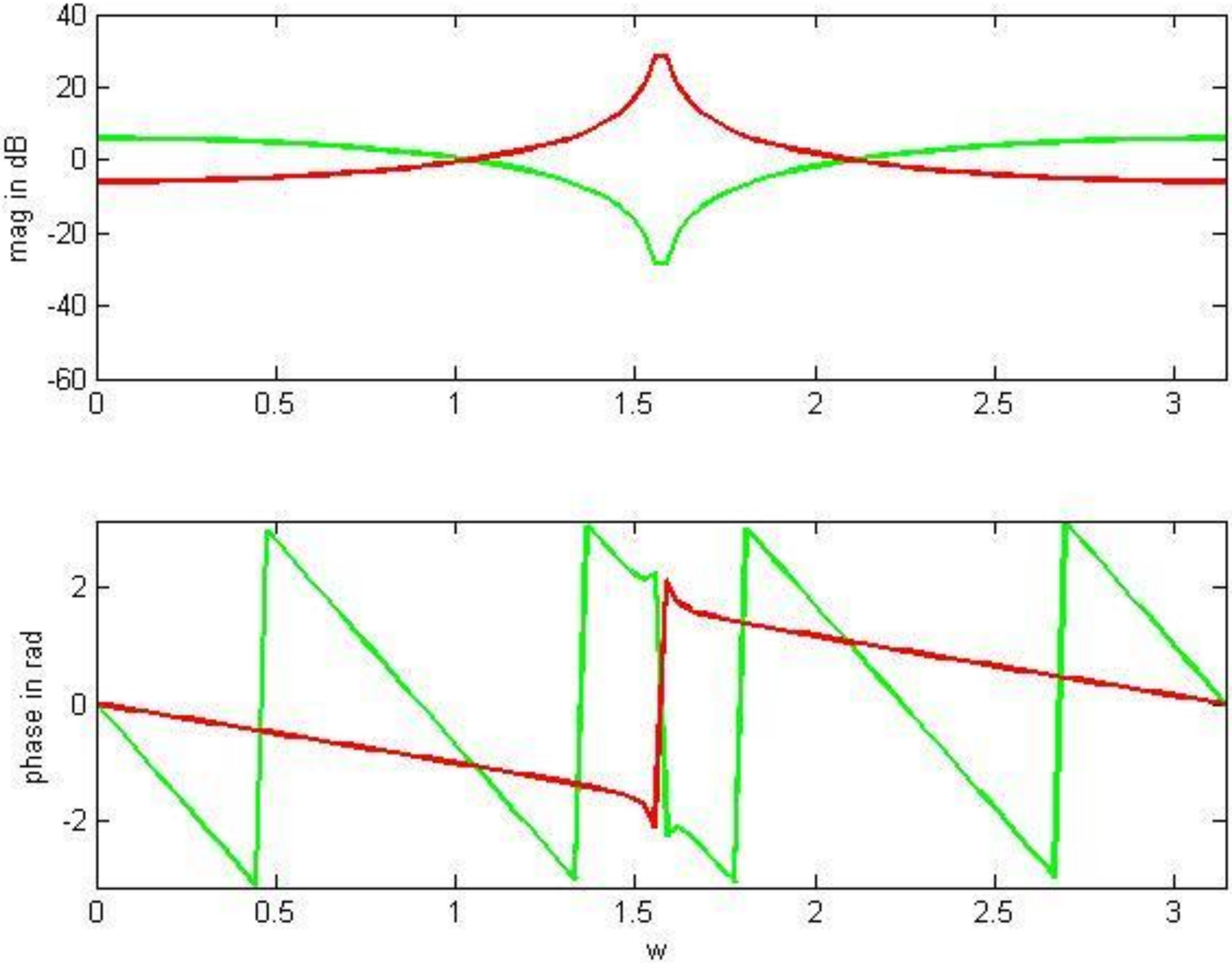
$$\min_{\theta} \frac{1}{m} \sum_{i=1}^{m} |C(e^{j\omega_i})E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}|^2$$

$$\text{subject to: } \max_{i=1,\dots,m} |C(e^{j\omega_i})E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}| \leq 1$$

Convex optimization program!

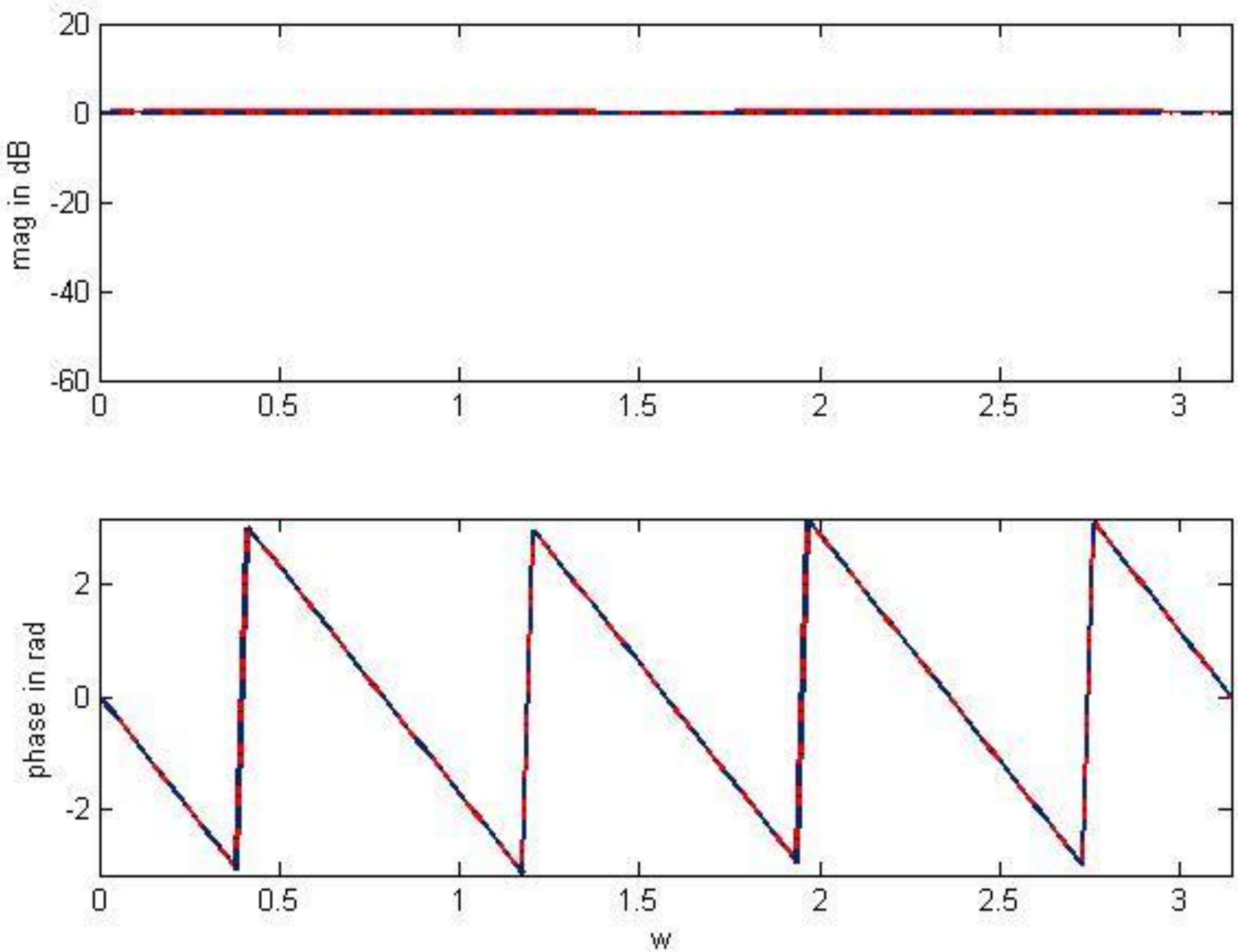
Channel equalization: problem solution

channel and equalizer [k=8, r=20]



Channel equalization: problem solution

cascade channel-equalizer vs. delay system [k=8, r=20]



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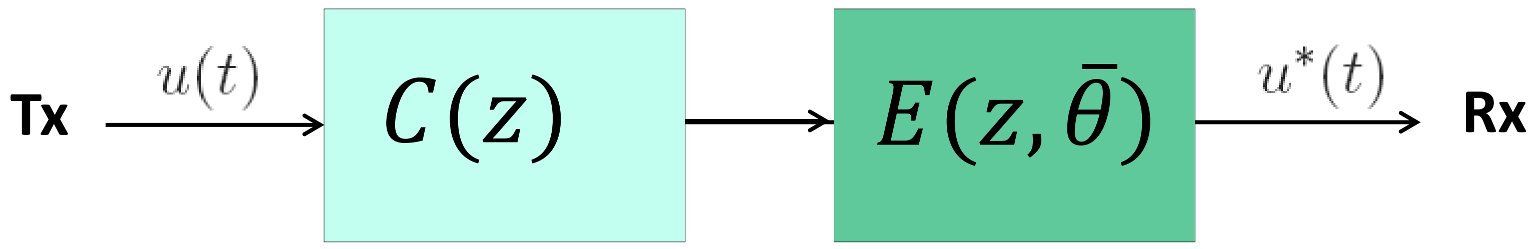

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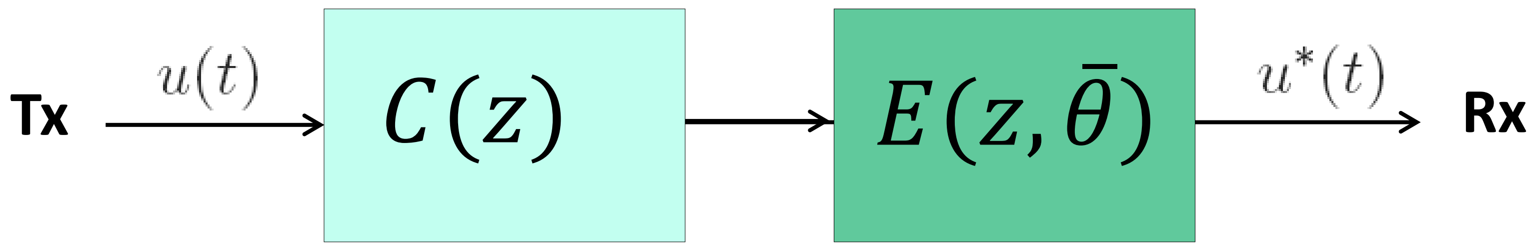

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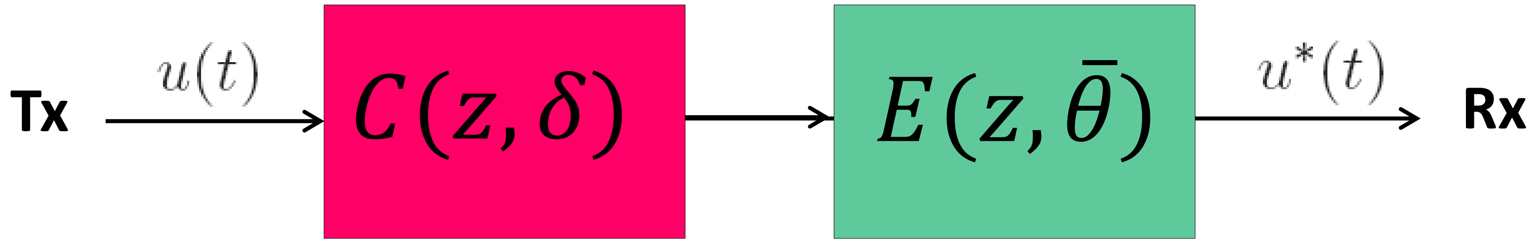
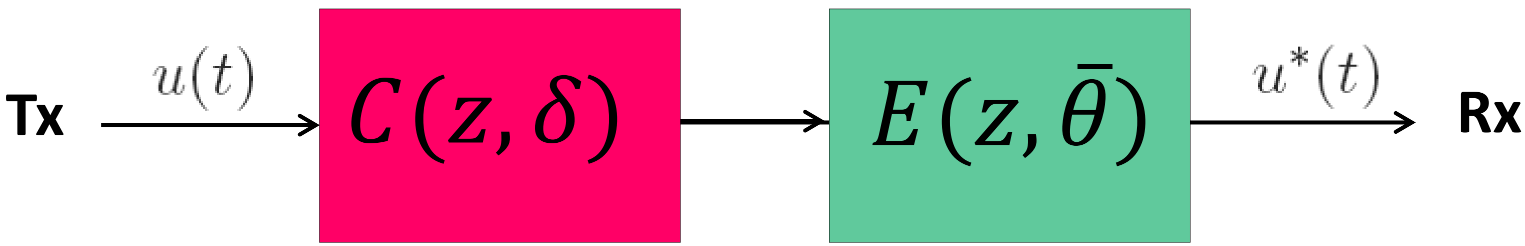


Example:
mobile communications, where the channel characteristics depend on the geographic position of the user

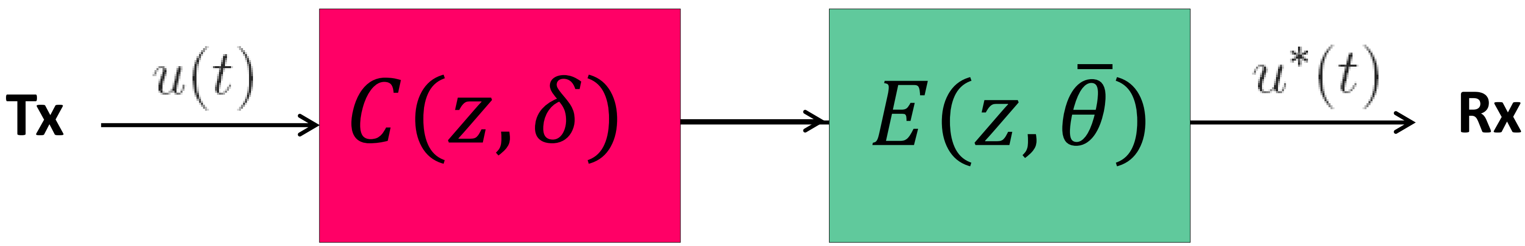
Channel equalization: problem solution

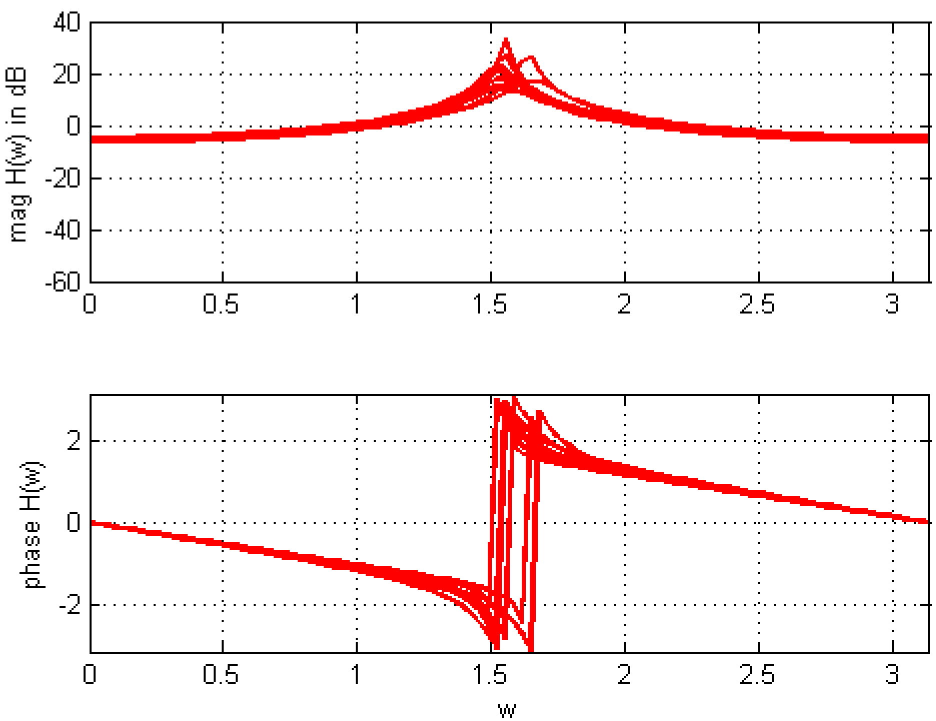


$$C(z,\delta) = \frac{1}{z^2 + \delta_1 z + \delta_2}$$

$$\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \in \underbrace{[-0.2,\ 0.2] \times [0.97,\ 0.99]}_{\Delta}$$

Channel equalization: problem solution

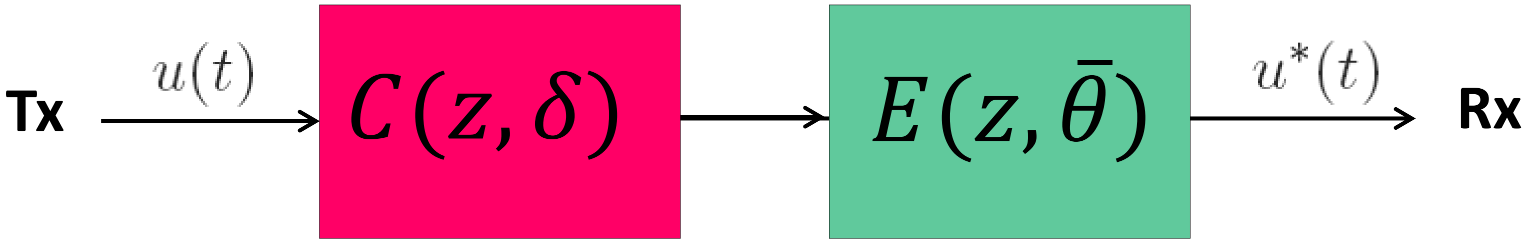




Possible scenarios

$$C(z,\delta_1)$$
$$C(z,\delta_2)$$
$$\vdots$$
$$C(z,\delta_N)$$

Channel equalization: problem solution



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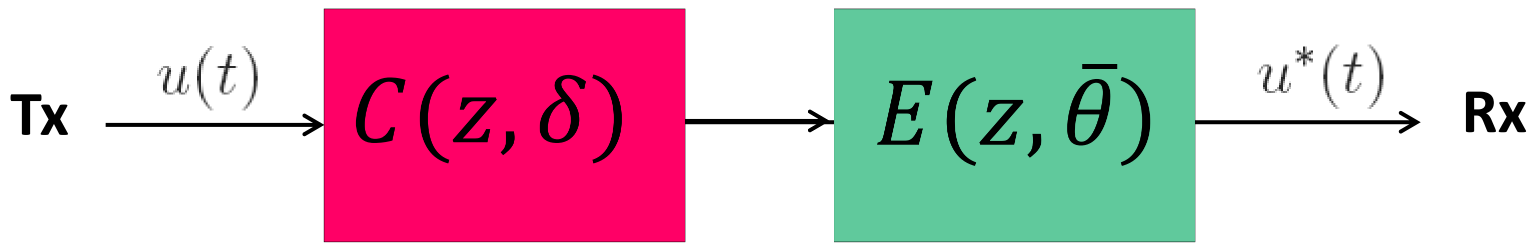

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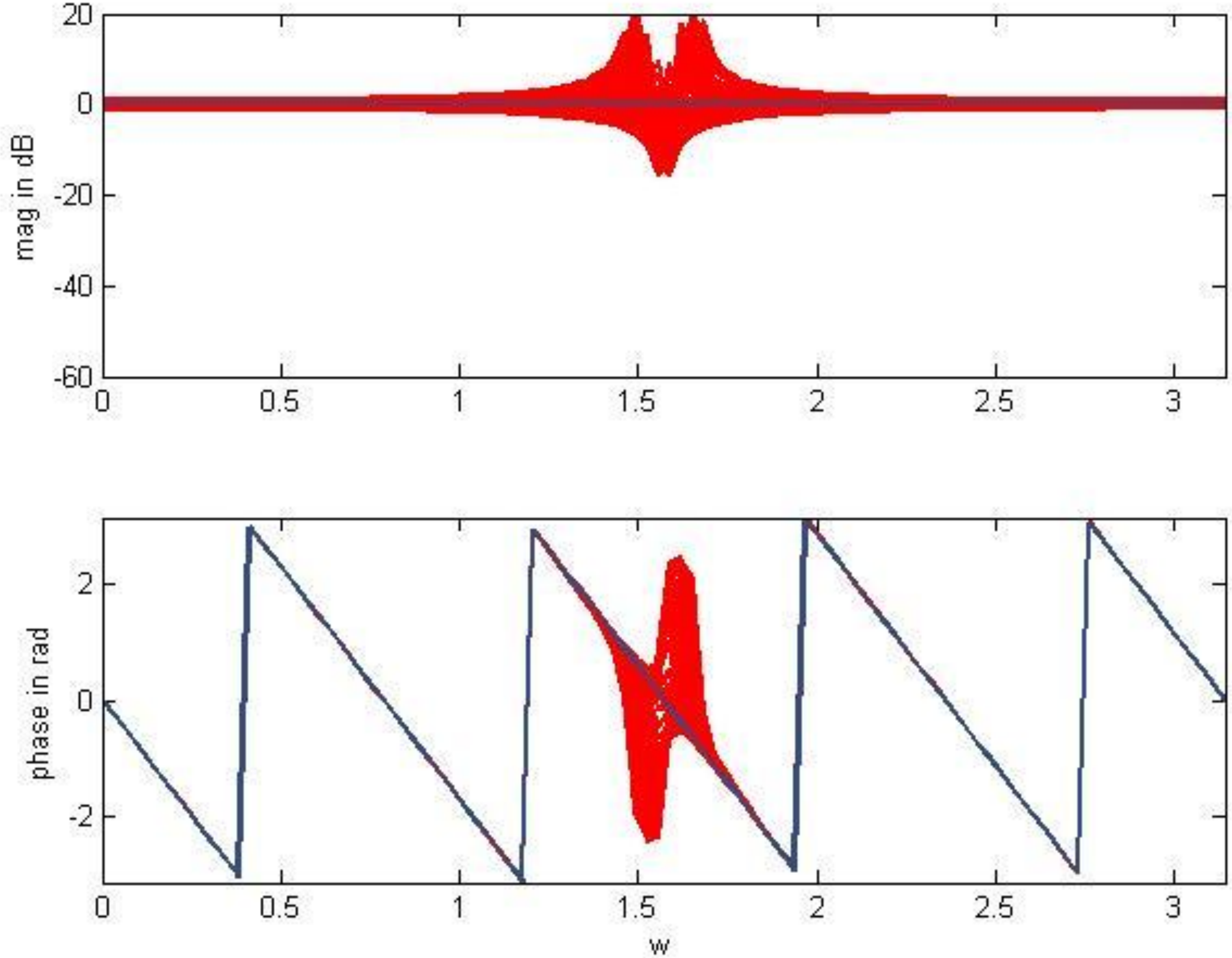

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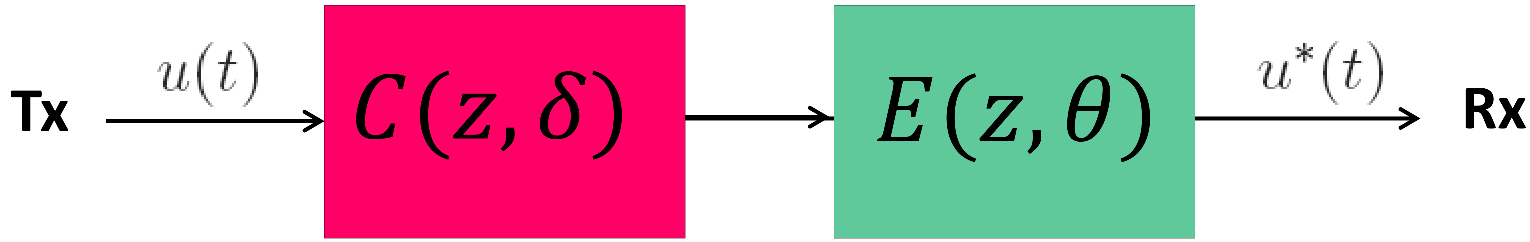


Robust approach

Determine the equalizer parameter vector θ so that

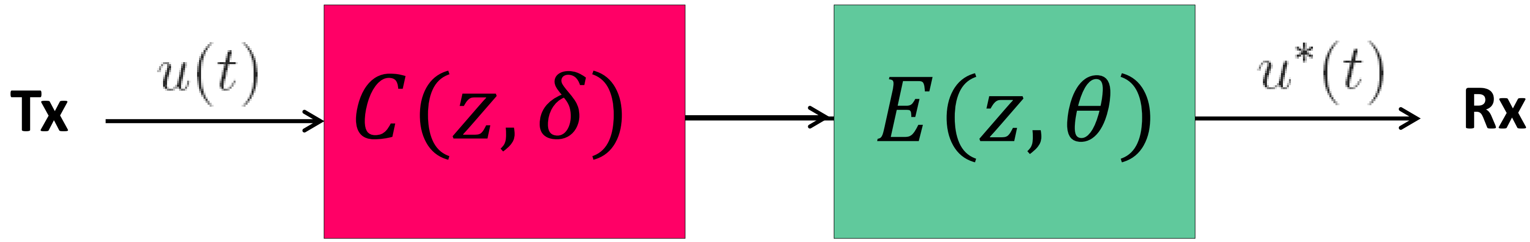
$$C(z,\delta)E(z,\theta) \simeq z^{-k},\ \forall \delta \in \Delta$$

Chance-constrained approach

Determine the equalizer parameter vector θ so that

$$C(z,\delta)E(z,\theta) \simeq z^{-k},\ \forall \delta \in \Delta_\varepsilon,\ P(\Delta_\varepsilon) = 1 - \varepsilon$$

Channel equalization: problem solution



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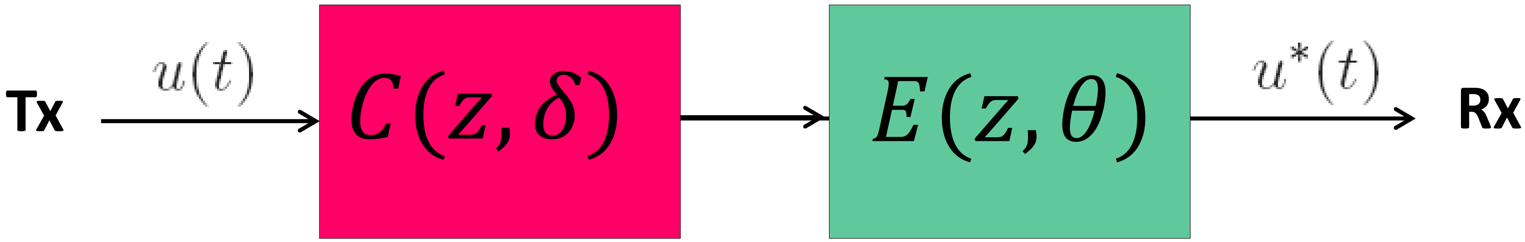
→ scenario solution

Channel equalization: chance-constrained solution

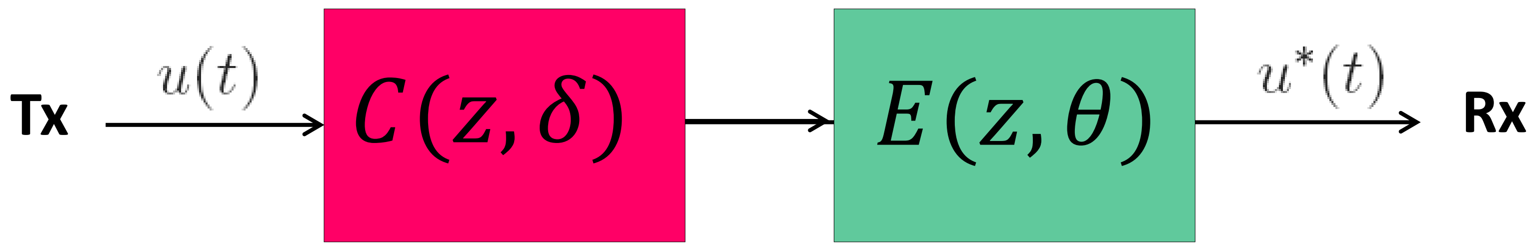


Chance-constrained optimization problem

$$\min_{\theta} \frac{1}{m} \sum_{i=1}^{m} |C(e^{j\omega_i}, \bar{\delta}) E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}|^2$$

$$\text{subject to: } P\left(\max_{i=1,\dots,m} |C(e^{j\omega_i}, \delta) E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}| \leq 1 \right) \geq 1 - \varepsilon$$

Channel equalization: chance-constrained solution



Chance-constrained optimization problem

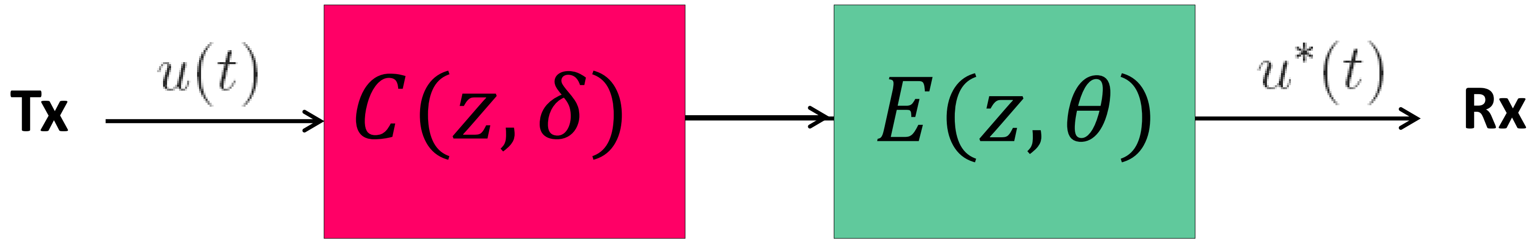
nominal channel

$$\min_{\theta} \frac{1}{m}\sum_{i=1}^{m} |C(e^{j\omega_i},\bar{\delta})E(e^{j\omega_i},\theta) - e^{-j\omega_i k}|^2 \qquad \bar{\delta} = \begin{bmatrix} 0 \\ 0.98 \end{bmatrix}$$

$$\text{subject to: } P\left(\max_{i=1,\dots,m} |C(e^{j\omega_i},\delta)E(e^{j\omega_i},\theta) - e^{-j\omega_i k}| \leq 1\right) \geq 1-\varepsilon$$

uniform distribution over Δ

Channel equalization: scenario solution



Scenario optimization problem

$$\min_{\theta} \frac{1}{m} \sum_{i=1}^{m} |C(e^{j\omega_i}, \bar{\delta}) E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}|^2$$

$$\text{subject to: } \max_{i=1,\dots,m} |C(e^{j\omega_i}, \delta^{(s)}) E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}| \leq 1, \;\; s = 1, 2, \dots, N$$

where $\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(N)}$ are independently extracted from Δ according to the uniform distribution

Channel equalization: scenario solution

What about chance-constrained feasibility of the scenario solution?

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$$\min_{\theta,h} h$$

$$\text{subject to:} \quad \max_{i=1,\ldots,m} |C(e^{j\omega_i}, \delta^{(s)})E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}| \leq 1, \;\; s = 1, 2, \ldots, N$$

$$\frac{1}{m}\sum_{i=1}^{m} |C(e^{j\omega_i}, \bar{\delta})E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}|^2 \leq h$$

The number of optimization variables is: d = r+1 = 21

If we set $\beta = 10^{-7}$ and $\varepsilon = 0.05$, then, we obtain N = 1063 from

$$\sum_{i=0}^{d-1} \binom{N}{i} \varepsilon^i (1-\varepsilon)^{N-i} \leq \beta$$

What about chance-constrained feasibility of the scenario solution?

$$\min_{\theta,h} h$$

$$\text{subject to: } \max_{i=1,\dots,m} |C(e^{j\omega_i}, \delta^{(s)})E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}| \leq 1, \;\; s = 1, \dots, 1063$$

$$\frac{1}{m}\sum_{i=1}^{m} |C(e^{j\omega_i}, \bar{\delta})E(e^{j\omega_i}, \theta) - e^{-j\omega_i k}|^2 \leq h$$

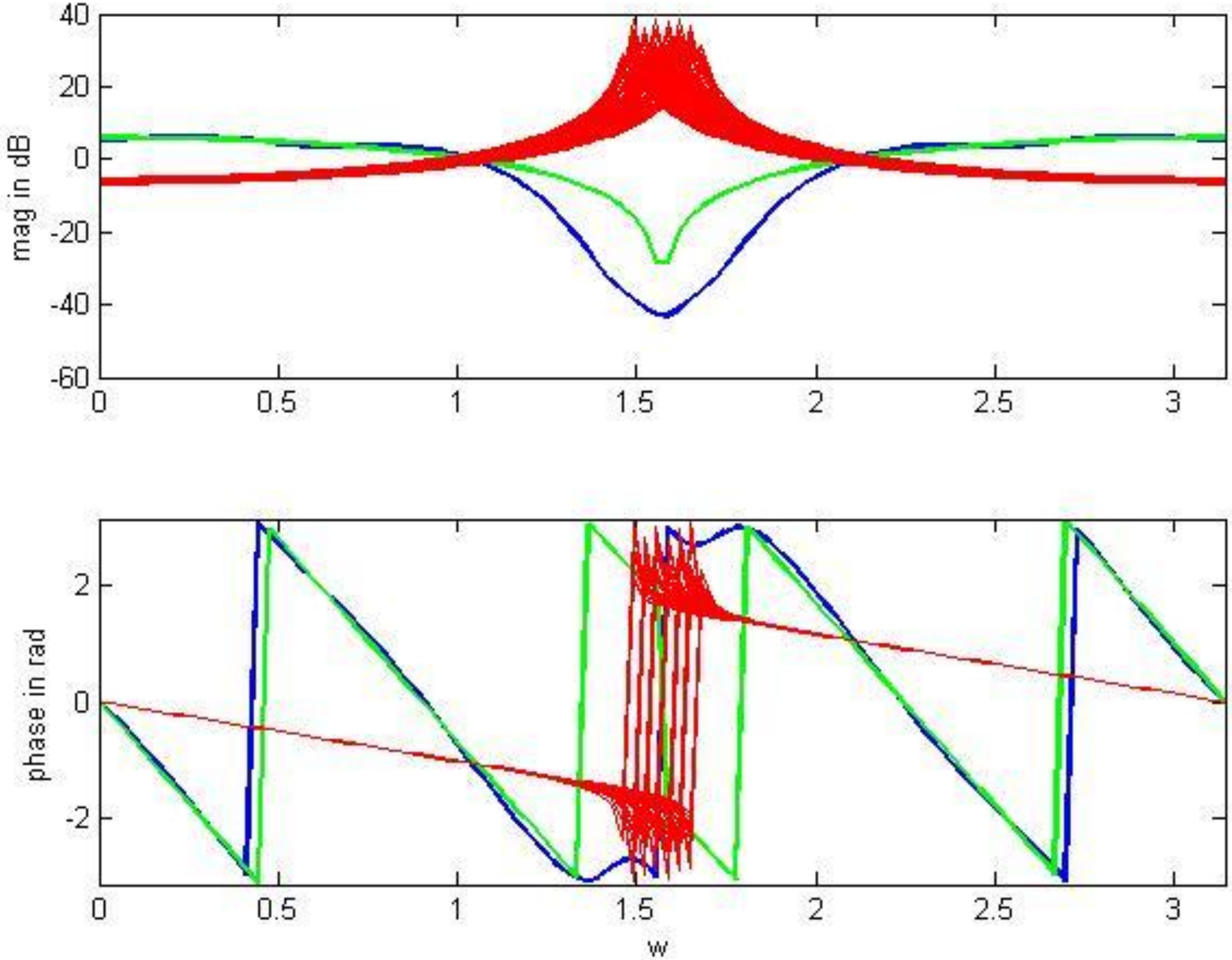
The scenario theory guarantees that the solution θ^* satisfies

$$P\Big(\delta \in \Delta : \max_{i=1,\dots,m} |C(e^{j\omega_i}, \delta)E(e^{j\omega_i}, \theta^*) - e^{-j\omega_i k}| \leq 1\Big) \geq 1 - \varepsilon$$

with probability $\geq 1 - \beta = 1 - 10^{-7} \simeq 1$

Channel equalization: scenario solution

channel, equalizer and scenario equalizer [k=8, d=20]



Channel equalization: scenario solution

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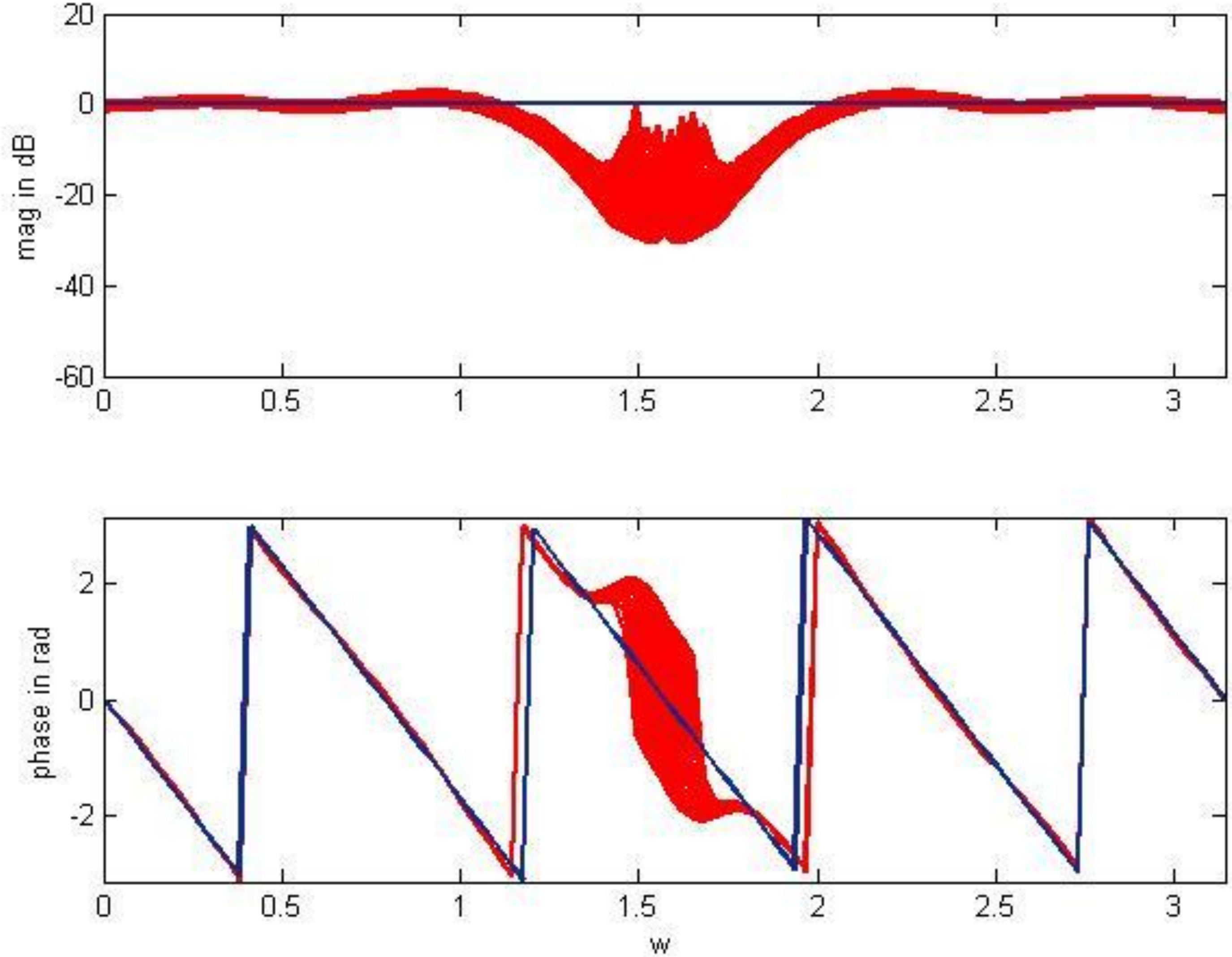
cascade channel-scenario equalizer vs. delay system [k=8, d=20]



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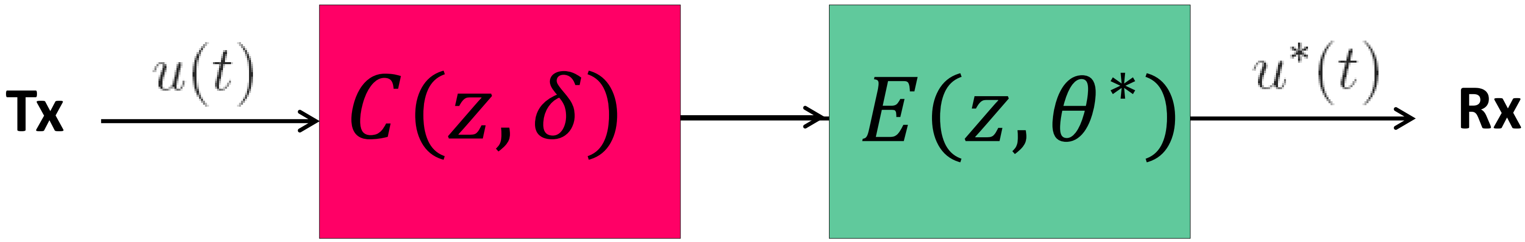

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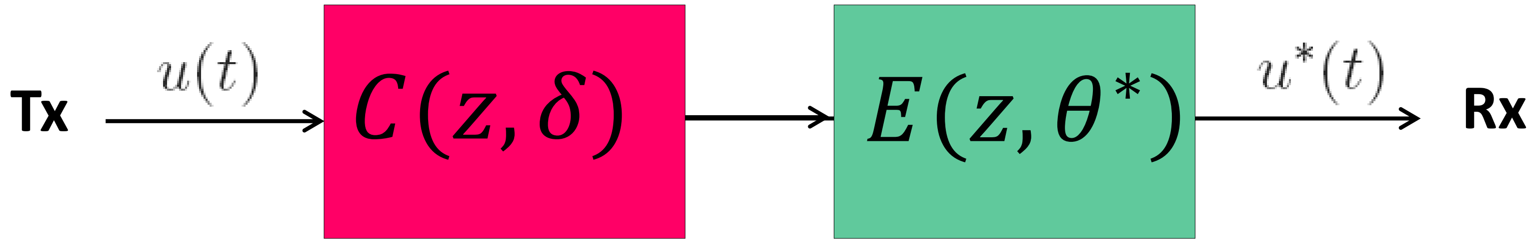

Applications of the scenario theory

Model reduction:

- M. Prandini, S. Garatti, R. Vignali.
  Performance assessment and design of abstracted models for stochastic hybrid systems through a randomized approach.
  Automatica, 2014. To appear.
- A.V. Papadopoulos and M. Prandini.
  Model reduction of switched affine systems: a method based on balanced truncation and randomized optimization.
  HSCC 2014, Berlin, Germany, April 2014.
- S. Garatti and M. Prandini.
  A simulation-based approach to the approximation of stochastic hybrid systems.
  ADHS'12, Eindhoven, The Netherlands, June 4-8, 2012.
- A. Abate and M. Prandini.
  Approximate abstractions of stochastic systems: a randomized method.
  50th IEEE CDC and ECC, Orlando, USA, Dec. 2011.

Game theory:

- D. Bopardikar, A. Borri, J. Hespanha, M. Prandini, M.D. Di Benedetto.
  Randomized Sampling for Large Zero-Sum Games.
  Automatica, vol. 49(5):1184-1194, 2013

Applications of the scenario theory

Constrained control design:

- L. Deori, S. Garatti, M. Prandini.
  Computational approaches to robust Model Predictive Control: a comparative analysis.
  IFAC World Congress 2014, Cape Town, South Africa, Aug. 2014.
- L. Deori, S. Garatti, M. Prandini.
  Stochastic constrained control: trading performance for state constraint feasibility.
  ECC 2013, Zurich, Switzerland, July 2013
- M. Prandini, S. Garatti, J. Lygeros.
  A Randomized Approach to Stochastic Model Predictive Control.
  51st IEEE CDC, Maui, Hawaii, Dec. 2012.

Approximate dynamic programming:

- A. Petretti and M. Prandini.
  An approximate linear programming solution to the probabilistic invariance problem for stochastic hybrid systems.
  53rd IEEE CDC, Los Angeles, USA, Dec. 2014.

Applications of the scenario theory

Building climate control:

- L. Deori, L. Giulioni, M. Prandini.
  Optimal building climate control: a solution based on nested dynamic programming and randomized optimization.
  53rd IEEE CDC, Los Angeles, USA, Dec. 2014.
- F. Borghesan, R. Vignali, L. Piroddi, M. Strelec, M. Prandini.
  Micro-grid energy management: a computational approach based on simulation and approximate discrete abstraction.
  52nd IEEE CDC, Firenze, Italy, Dec. 2013.
- F. Borghesan, R. Vignali, L. Piroddi, M. Strelec, M. Prandini.
  Approximate dynamic programming-based control of a building cooling system with thermal storage.
  IEEE ISGT 2013, Copenhagen, Denmark, Oct. 2013.

Applications of the scenario theory

Reserve scheduling:

- V. Rostampour, K. Margellos, M. Vrakopoulou, M. Prandini, G. Andersson, J. Lygeros. Reserve Requirements in AC Power Systems with Uncertain Generation. IEEE ISGT 2013, Copenhagen, Denmark, Oct. 2013.
- K. Margellos, V. Rostampour, M. Vrakopoulou, M. Prandini, G. Andersson, J. Lygeros. Stochastic unit commitment and reserve scheduling: A tractable formulation with probabilistic certificates. ECC 2013, Zurich, Switzerland, July 2013.

Aerospace applications:

- A. Falsone, F. Noce, M. Prandini. A randomized approach to space debris footprint characterization. IFAC World Congress 2014, Cape Town, South Africa, Aug. 2014.
- Y. Yang, J. Zhang, K. Cai, M. Prandini. A stochastic reachability analysis approach to aircraft conflict detection and resolution. 2014 IEEE MSC, Antibes, France, Oct. 2014.